CRITIQUE OF WARD SILVER's Nov. 2019 QST ARTICLE

on

TRANSMISSION LINE TRANSFORMERS

and

An INTRODUCTION to SMITH CHARTS

By

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Use transmission lines to convert impedances.

H. Ward Silver, NØAX

In the previous article,[1] "About Impedance-Matching Circuits," we discussed what impedance is, and learned that what we call "impedance matching" is really "impedance converting." We reviewed several circuits and components that perform impedance conversion.

This article covers other methods of converting impedance using the properties of transmission lines. Special lengths, combinations, and connections of feed lines can be used. At antenna feed points, the same ideas can be used to create structures that convert impedances, too.

Feed Lines and Impedance

We start with a feed-line *characteristic impedance*, abbreviated Z_0. This tells us what voltage and current are created by power flowing in a feed line. Just like in a circuit, Z_0 tells us the ratio of voltage and current, but this time, in the line.

To experience a mechanical analogue of characteristic impedance, get a small-diameter coffee stirrer and a large-diameter drinking straw. Blow a short, sharp puff of air through each. Even though you blow equally hard into each (voltage) you get only a little air through the small tube (current), but a lot of air gets through the large one. The small tube has a higher characteristic impedance to pressure waves.

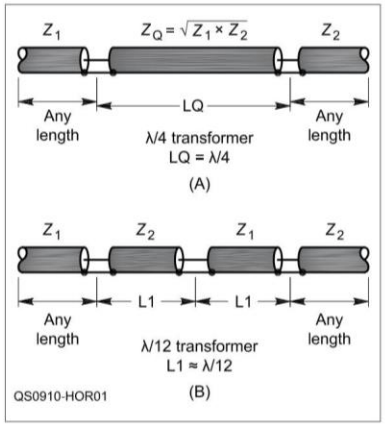


Figure 1 — The quarter-wave and 1⁄12-wave synchronous transformers. The series of reflections created by the impedance mismatches at each end of the matching sections creates the impedance conversion.

Back to RF — if power flowing in a feed line encounters some impedance other than Z_0, such as a different type of feed line, the voltage and current change abruptly at the junction. To create this change, the initial wave splits into two; one in the new feed line and one reflected back toward the source in the original feed line. Just like water waves reflecting from a wall or rock, a pattern of interference is created between the incoming *forward* wave and the

"Just like in a circuit, Z_0 tells us the ratio of voltage and current, but this time in the line."

reflected wave. Even though both waves are moving, the voltage and current interference patterns are stationary, called a *standing wave*. The *standing wave ratio* (*SWR*) is the ratio of the maximum to minimum voltage or current in the pattern. See the sidebar, "Explaining SWR in Black and White."

Hams are counseled to minimize SWR, because standing waves created by the reflected power have higher peak voltages — increasing *dielectric loss* — and higher peak currents — increasing *resistive loss*. Feed line loss in general is dissipation of the RF signal as heat by any means; dielectric loss occurs in the insulation between the center conductor and shield, caused by the ac voltage between the conductors. Resistive loss is caused by the resistance of the center conductor and shield to ac current, and is proportional to the square of the current. As frequency increases, so do both dielectric and resistive losses.

In addition, after the reflected power makes its way back to the source, such as a transmitter, it can be reflected again, heading back to the load or the antenna. Each unwanted extra trip through the feed line results in loss, until all of the power is either transferred to the load or dissipated as heat. To avoid loss, we try to match the line and load impedances, minimizing reflections and SWR.

Refer to November 2019 QST for Full Article by N0AX

In the following we will explain the theory behind ward's Figure 1(A) and (1B)

Lossless line equations showing voltage, current & impedance along a transmission line.

We shall write the equations in terms of distance d from a load, Z_load, located at d = 0 with +d pointing back towards the generator and the assumed, common, single frequency time variation suppressed.

V(d) = V_load*cos(βd) +jI_load*Z0*sine(βd)

I(d) = I_load*cos(βd) +j(V_load/Z0)*sine(βd)

Where V_load, I_load, V(d) and I(d) are complex phasors in general with magnitude & phase and

Z0 is the transmission line impedance, typically 50 to 600 Ohms & β = 2*pi/λ.

Recall from H.S. trig: cos(0) = cos(360) =1; cos(90) = 0; cos(180) = -1; cos(270 = 0)

and sine(0) =sine(360) = 0; sine (90) = 1; sine(180) = 0 and sine(270) = -1

The complex impedance, Z(d), seen by a generator at distance d from the load, is hence:

Z(d) = V(d)/I(d) which after dividing N and D by cos(βd)

and noting V_load/ I_load = Z_load we finally get

Z(d) = Z_load*[1 + j(Z0/Z_load)*tan(βd)]/[1+j(Z_load/Z0)tan(βd)]

where tan(βd) = sine(βd)/cos(βd)

Now back to the Ward Silver QST article. Consider his Quarter Wave Line

Here is what happens if we let βd = pi/2 (a quarter wavelength line) in the line equations for the impedance seen at the output of the quarter wave line:

tan(βd) -> 1/0 = infinity so we must approach gradually to see what happens. Here tan(βd) is just assumed very large so it dominates in the Numerator and Denominator in the expression for Z(d) compared to the leading ones.

Then

Z(d=> λ/4) => Z_load*[j(Z0/Z_load)*tan(βd)]/[j(Z_load/Z0)*tan(βd)]

After a lot of cancellations, we obtain:

Z(d=> λ/4) = Z_load*(Z0/Z_load)/(Z_load/Z0)

= Z0* Z0/Z_load = $Z0^2$/Z_load

the desired result which can be rewritten as

$Z0^2$ = Z(λ/4)*Z_load   or Z0 = SQRT(Z(λ/4)*Z_load)

In N0AX's QST article Z0 is called Z_0, Z(λ/4) is Z_1 and Z_load is Z_2.

Note, the same transformation happens for d = any odd multiple of λ/4

Interesting!

Half Wavelength Line Analysis

Next here is what happens if we let d = n* λ/2 (distance any multiple of a half wavelength)

Then βd = (2*pi/ λ)*(n* λ/2) =n*pi

And since tan (n*pi) = 0 for all integer values of n

Z(d = n* λ/2) =

Z_load*[1 + j(Z0/Z_load)*tan(n*pi)]/[1+j(Z_load/Z0)tan(n*pi)]

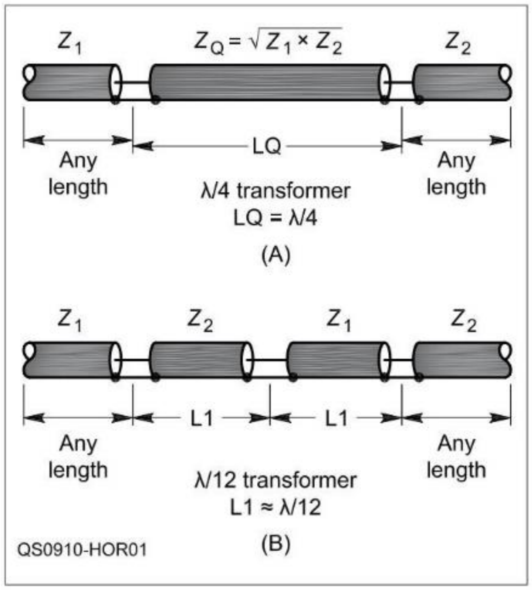
= Z_load (1+ j(Z0/Z_load)*0]/[1+j(Z_load/Z0)*0)]

= Z_load *1/1   = Z_load

Thus, the load impedance always repeats every half wavelength away from the load regardless of the transmission line impedance.

Also Interesting!

Consider the 1/12 wavelength impedance transformers Silver shows in his Figure 1(B).



The benefit here that no odd-ball impedance matching line is required. All lines are Z_1 or Z_2.

A typical example would be matching a 400 Ohm line(Z_2) to a 50 Ohm line (Z_1) using transmission lines assuming the load (an antenna typically) is matched to the 400 Ohm line.

Working from right to left in Wards Figure 1B we would have $Z_1(0)$ = Z_antenna = 400 Ω and we want to show $Z_2(L1)$ = 50 Ω which matches the left coax.

While we could use the basic line equations twice to show **(or not show as I claim)** that two 1/12 wavelength lines can accomplish this match that would involve a lot of complex number arithmetic by hand or on a computer.

Instead let's try using a Smith Chart to obtain a graphical solution and show that this choice of cascaded 1/12 wavelength lines does (or does not) transform 400 Ohms to 50 Ohms. (Smith Charts are on the Extra Class exam so this is good practice.)

The Complex Reflection Coefficient & the Smith Chart

Smith Charts were invented by Peter Smith in 1939 as a graphical alternative to solving the complex transmission line equations before digital computers.

The Smith Chart graphs the complex reflection coefficient, Γ, in terms of distance d from the load (antenna) or conversely distance s from the source (transceiver) where Gamma is a complex number with real part u and imaginary part v. (I shall only use distance, d, from the load here.)

Specifically, Γ is the ratio of a forward propagating voltage wave on the line to a reverse propagating or reflected voltage wave on the line at any distance d from the load (antenna). (The figure below shows the coefficient at the load but it can be measured anywhere along the line.)

- V_i is incident voltage.
- V_r is reflected voltage.

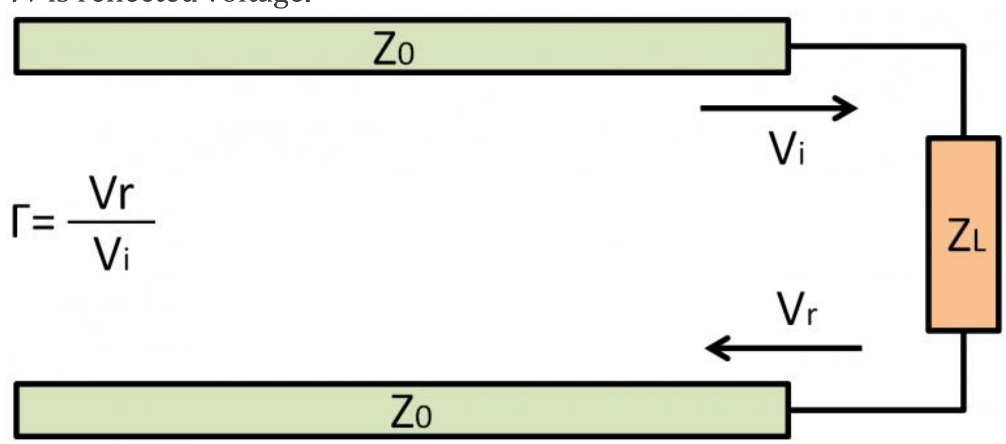


Fig. 2   Reflection coefficient (Γ)

Magnitude of Reflection Coefficient & Relation to SWR

As noted, gamma is a complex number that changes along the line but its magnitude, $| \Gamma |$, anywhere along the line is constant if we neglect losses.

Thus

$\Gamma (d) = u(d) + jv(d)$

but $| \Gamma |$ = square root $(u^2 + v^2)$ = constant for any d.

Now it can be shown that there is a simple relationship between the magnitude of the reflection coefficient and the SWR, the latter which is also constant for all d neglecting losses. Namely,

$| \Gamma | = (SWR - 1)/(SWR + 1)$

Equivalently

$SWR = (1 + | \Gamma |)/(1 - | \Gamma |)$ = constant

The Reflection Coefficient's Phase angle

In general as one moves away a distance d from the load Γ acquires a non-zero phase angle even if the load impedance were also a pure resistance like Z0.

To see this refer to the proceeding figure again but assume we are a distance, d, away from the load.

Then the incident wave is still a distance d from the load while the reflected wave has traveled up to and back from the load. Defining the wavenumber $\beta = 2*pi/\lambda$ where lambda is the wavelength this means that the reflected wave gains a phase angle of $2*\beta*d$ relative to the incident wave as one moves back from the load.

Thus

$\Gamma(d) = \Gamma(0)*\exp(-2*\beta*d)$ using phasor notation

Where Γ(0) is by definition Γ_load, the reflection coefficient at the load. The latter is itself also a complex number unless Z_load is a pure resistance.

There is also a relationship between $\Gamma(d)$ and the line impedance $Z(d)$

At the load $\Gamma(d) = \Gamma(0) = u(0) + jv(0) = \Gamma_{Load}$

It can also be shown for any load impedance Z_load that

$\Gamma_{Load} = (Z\_load – Z0)/(Z\_load + Z0)$

Similarly, anywhere along the line we have

$\Gamma(d) = (Z(d) – Z0)/(Z(d) + Z0)$

Where $Z(d) = R(d) + jX(d)$

We shall normalize Z(d) by Z0. Denote this normalization with a prime.

Then $Z'(d) = R(d)/Z0 + jX(d)/Z0 = r(d) + jx(d)$

where r and x are the normalized resistance and reactance at d from the load.

Similarly, dividing both the numerator and denominator on the right hand side of the expression for $\Gamma(d)$ by Z0

$\Gamma(d) =u(d) + jv(d) = (Z'(d) – 1)/(Z'(d) + 1)$

Substituting for Z'(d) we now have a relationship between u, v and r,x namely

$u(d) + jv(d) = (r(d) – 1 +jx(d))/ (r(d) + 1 +jx(d))$

Relationship between $\Gamma(d)$ and the line impedance Z(d) continued…

From all of this mathematics we have obtained a complex variable equation relating the real and imaginary components of Γ, namely u and v respectively, to the real and imaginary parts of the normalized impedance Z', namely r and x respectively, at any point along the line.

Repeating this key relationship in terms of these 4 quantities we have:

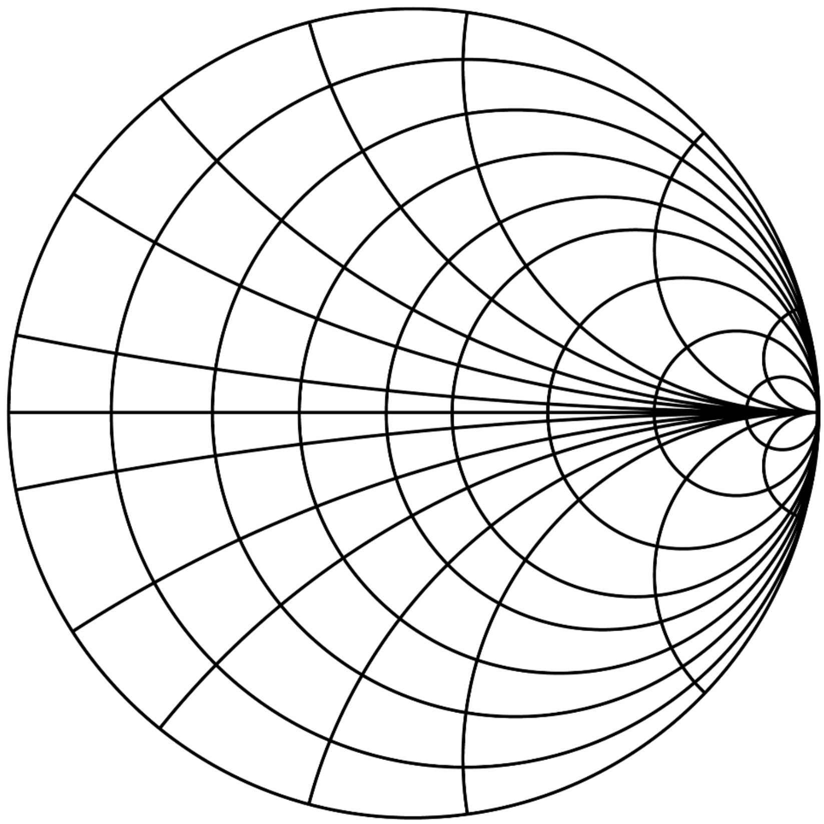
$$u(d) + jv(d) = (r(d) - 1 + jx(d))/ (r(d) + 1 + jx(d))$$

We actually have two scalar equations since the real parts of both sides of the equal sign must be equal and the imaginary parts of both sides must be equal.

This means that any r,x pair in the Z' plane will map into a corresponding u,v pair lying within the unit circle in the Γ plane. Actually, there are multiple solutions but we throw out the non-physical ones corresponding to negative resistances r.

This is useful for plotting the evolution of the line impedance as one moves a distance d from the load because, unlike the line impedance Z or the normalized line impedance Z', **the magnitude of Γ is bounded by unity**. Further **any value of normalized input impedance Z' will map into a unique value of Γ inside the unit circle** (no need for a huge sheet of linear graph paper to cover all physically realizable cases for analyzing typical "ham" antennas).

THE SMITH CHART SOLVES THE LOSSLESS LINE EQUATIONS GRAPHICALLY FOR Z'(d)



Constant resistance and reactance circles plotted together

In the Smith Chart circles are constant r lines where r(d) = R(d)/Z0 and arcs are constant x lines where x(d) =X(d)/Z0. Really the arcs are also circles but only the portions with the unit circle correspond to physically realizable resistances normalized or unnormalized.

The Smith chart expresses d in fractions of λ along its periphery. A half wavelength distance (d = λ/2) corresponds to one full rotation around the periphery of the chart.

To use a Smith Chart to show how a load impedance varies along a line:

Plot Z'(0) on the chart as a point. Draw a vector from the center of the chart (r=1, x=0 point) to Z'(0).

Extend the vector to the chart periphery to determine how Z' will change with distance from the load, d.

Relative movement of the extended vector clockwise on the outer periphery denotes movement by a distance d toward the generator (transceiver here).

Using a protractor set to the length of our initial normalized impedance vector, **rotate** the vector by the relative distance d along periphery in wavelengths.

See where the original length vector falls inside chart. This is the normalized load impedance at distance d. In other words this is impedance a load at d = 0 would present to an observer located at distance d feeding the load.

To compute what impedance the measured impedance Z_gen at the generator would present at some distance **d from the generator** as we travel back towards the load (antenna) we would rotate in the **opposite** direction, i.e. counterclockwise, on the Smith chart periphery. This is useful in the practical case where one measures the impedance at the generator/transmitter and wants to know what impedance the antenna is actually presenting to the transmission line. (The total line length would be d then.) We will not need this here but it shows another use of the Smith Chart. There are lots more uses.

Now back to Ward Silver's cascaded 1/12 wavelength transformers:

In the article we will need to use the Smith Chart **twice** to get the final impedance presented to the 50 Ohm line back to the transmitter at the output of the 400 Ohm λ /12 section, Z_2.

It is assumed that the antenna was properly matched to line Z_1 so the length of that connecting line length does not matter.

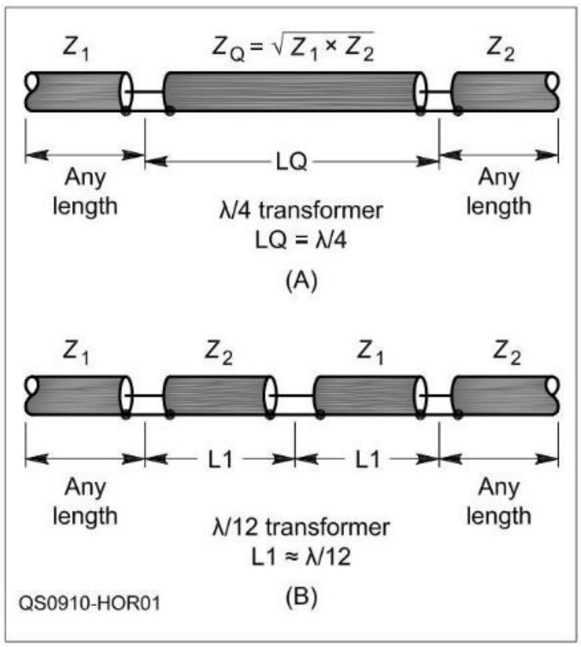


Figure 1 — The quarter-wave and 1⁄12-wave synchronous transformers. The series of reflections created by the impedance mis-matches at each end of the matching sec-tions creates the impedance conversion.

Let's pull up a complete Smith Chart and try doing a double application of $\lambda/12$ = 0.0833λ sections.

The first section is 50 Ohms, the second is 400 Ohms.

The normalized "load" input to Z1 namely $Z_1'(0)$ = [400+ j0]/50 = 8 + j0 Ohms.

The renormalization for the second, 400 Ohm, section amounts to taking 50/400 = 1/8 of the normalized output impedance from the first section as the "load" or starting impedance of the 2nd Section.

We draw a vector from the chart "origin" at 1 +j0 to the load impedance of 8 + j0

Rotating this vector by 0.0833 wavelength clockwise around the periphery corresponding to the, Z1, the first 1/12th wavelength section, we land on an intermediate normalized impedance of ~ 0.42 – j1.44.

We multiply the normalized intermediate impedance by 50/400 to get the new normalized "load" impedance for the second section, $Z_2(0)$. This comes out to be 0.0525 – j0.18.

We plot this point on the chart, draw a vector from the origin to 0.025 – j0.2 and again rotate **this** vector clockwise by another 0.08333 as we continue to move left toward the source in Figure 1B.

Hopefully this puts us on 50/400 + j0 for the normalized impedance at the far left side of the Z_2 section. Un-normalizing by multiplying by 400 we should get the "load" impedance presented to the 50 Ohm coax back to the transceiver source to be 50 + j0 ohms. That would be a perfect match.

Problem is my result was nowhere near a match to the 50 Ohm line to the generator. Thus, there is doubt cast on the 1/12 wavelength transformer approach to impedance matching, at least in my opinion.

Working backwards for this 50 to 400 ohm transformation example W8IMA, Richard, found out the two sections should be 0.051 wavelength sections (~ 1/20 wavelength) rather than 1/12 (0.083) wavelength sections.

Bottom line, the proper lengths L1 and L2 need to be tuned to the desired match!

Thus, the magic 1/12 wavelength line transformer design is a hoax!

Richard is sending our results to Ward Silver at QST

Anyone want to point out what we are missing?

Ron

K8DMR

Suggested Site: https://www.will-kelsey.com/smith_chart/#

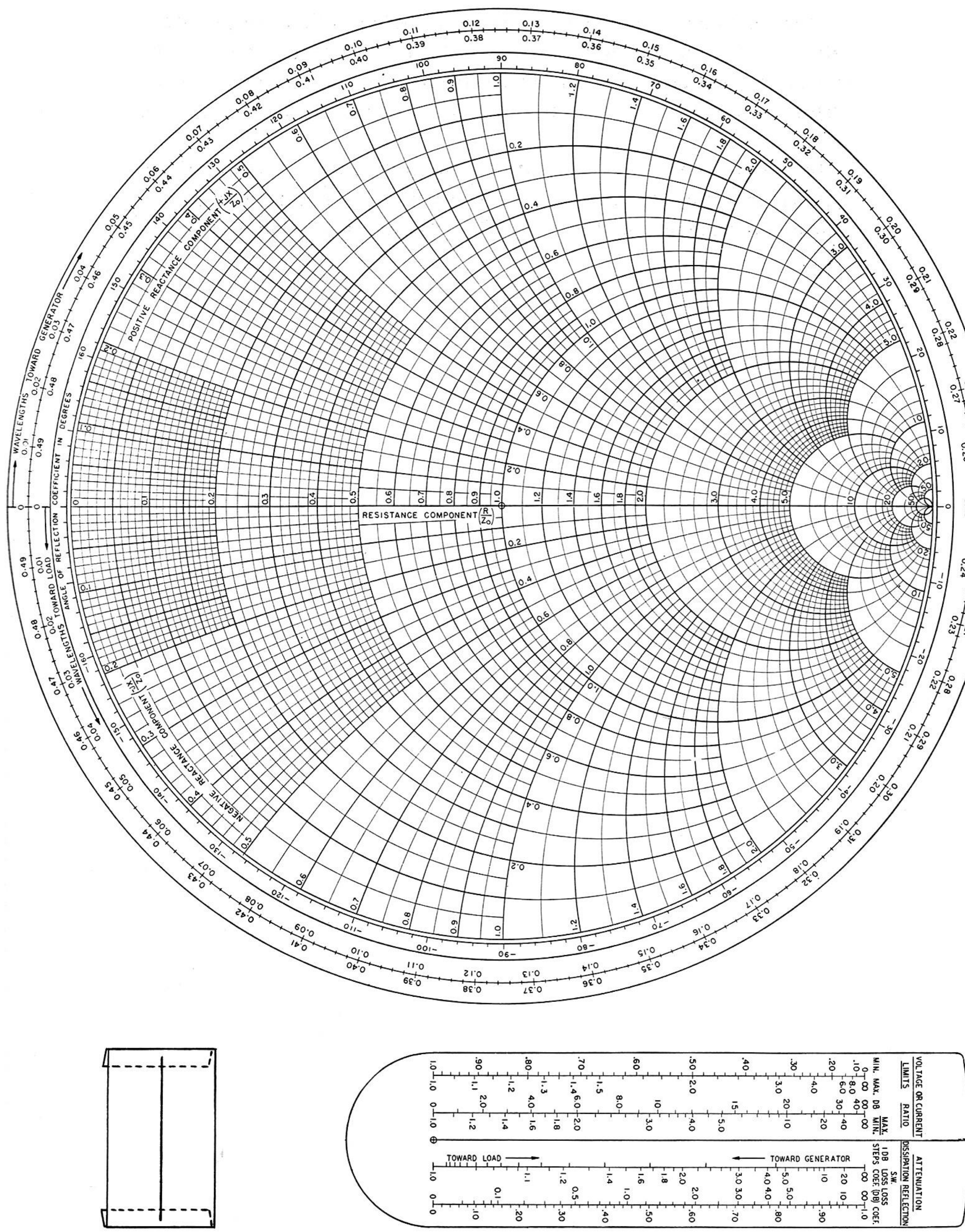


Fig. I.4. Improved transmission line calculator. (*Electronics, January, 1944.*)